EXPERIENCE BASED SEISMIC EQUIPMENT QUALIFICATION IN THE ASME-QME STANDARD: EQUIPMENT CLASS DATABASE SIZE REQUIREMENTS

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ABSTRACT

In the early 1980’s the Seismic Qualification Utility Group (SQUG) was formed to develop a generic methodology to disposition Unresolved Safety Issue (USI) A-46. Working in conjunction with the regulatory authorities and industry, SQUG developed a methodology and procedure to apply earthquake experience data to demonstrate the seismic ruggedness of electrical and mechanical equipment for resolution of USI A-46. In the early 1990’s, the ASME and IEEE formed a joint working group to investigate whether earthquake experience based equipment qualification could explicitly be incorporated into ASME QME-1 and IEEE-344. The joint ASME-IEEE working group concluded that experience based rules could be introduced into IEEE-344 and ASME QME-1. In response, the ASME QME Main Committee formed the Subgroup on Dynamic Qualification (SGDQ) to implement the recommendation of the joint ASME-IEEE Special Working Group. The Subgroup recently completed this effort and the QME-1 standard will include a prescriptive methodology to apply actual earthquake experience to the seismic qualification of mechanical equipment. As part of these changes, the QME-1 Standard provides requirements on equipment class database size for estimating seismic capacity based on earthquake experience data. This paper provides the technical basis for the required equipment class sample size and the associated reduction factors required for smaller sample sizes for using earthquake experience data.

Introduction

Section QR-A7422 specifies a minimum of 30 independent items that performed satisfactory to define an equipment class. Also in that section it provides Table QR-A7422-1 “Reduction Factors” for cases where there is less than 30 independent items. Depending on the number of independent items, a reduction factor is selected per the table and then multiplied times the earthquake experience spectrum (EES) of QR-A7412 to produce a EES that has the same statistical confidence level as a reference active mechanical equipment class comprising 30 independent items. The following is the technical basis for the sample sizes and reduction
factors for the number of independent items for use in estimating equipment seismic capacity using earthquake experience data.

Sample Size and Reduction Factors

Let the average spectral capacity of a given equipment class, defined as a 5% damped spectral acceleration value averaged over the 3-8 Hz frequency range, be represented by the random variable $C$. The distribution of $C$ is taken as lognormal with a known (assumed) log-normal standard deviation, $\beta_c$, but an unknown lognormal mean, $\ln(C)$, where $C$ represents the median capacity.

Let the average spectral demand that the equipment class has been subjected to, defined as a 5% damped free-field spectral acceleration value averaged over the 2.5-8 Hz frequency range, be represented by the random variable $D$. The distribution of $D$ is taken as log-normal with a known (assumed) log-normal standard deviation, $\beta_D$, but an estimated lognormal mean, $\ln(D)$, where $D$ represents the median demand.

Next consider $n$ independent equipment items from the equipment class, with known free-field spectral demand $\{D_1, \ldots, D_n\}$ resulting in an average Reference Spectrum value, $D_{ave} = RS$. Each of the $n$ items has survived the respective input motion represented by $D_i$ without damage. Caveats are used in defining the equipment class to exclude items with damage due to non-engineered attributes such as lack of anchorage or inadequate restraint.

The ratio of capacity to demand, $C_i/D_i$, for all $n$ items is greater than unity, or

$$C_i/D_i > 1,$$

since no damage has been observed in any of the $n$ equipment items belonging to the equipment class.

The ratio of spectral capacity to spectral demand, $X = C/D$, is also a lognormal variable with mean $\ln(X) = \ln(C/D)$ and log-normal standard deviation $\beta_X = \{(\beta_D)^2 + (\beta_C)^2\}^{1/2}$. The probability of failure for an item of equipment is given by

$$P_f = P(X<1) = F(X=1),$$

where $F$ is the cumulative distribution function (CDF) of $X$.

If a reduced variate $u$ is defined as $u = \ln(X)/\beta_X$, $u_0 = \ln(X)/\beta_X$, then

$$F(X) = \Phi(z),$$

where $z = u - u_0$ and $\Phi$ is the normal CDF. Thus,

$$P_f = F(X=1) = P(u<0) = P(z<-u_0) = \Phi(-u_0)$$

The probability of survival for an equipment item is

$$P_s = 1-P_f.$$
Now, given \( n \) pairs of independent \( D_i, C_i \) with known \( D_i \) and average RS but unknown \( C_i \), apply the constraint, \( X_i = C_i/D_i > 1 \), since no failure has been observed in the \( n \) equipment items. If the \( X_i \) are ordered such that \( X_1 < X_i < X_n \), the minimum probability of survival is given by

\[
P(X_i > 1) = \prod_i \{1-F(X_i)\} \quad X_i = 1 = (1-P_F)^n.
\]

Since \( C \) is unknown, it can only be specified by the assignment of a confidence coefficient. The lower confidence limit on \( P_F \) is found by considering the probability of an assumed failure for an \((n+1)\)th item of equipment. This probability of failure is taken as the confidence level, \( \gamma \), such that the observed result of \( n \) cases of no failure is the best that could have occurred. Thus,

\[
\gamma = 1 - (1-P_F)^{n+1}
\]

is the probability of failure for at least one item given the survival of \( n \) items.

Now the population mean, \( \ln(X) \), which assures that, for a given level of confidence \( \gamma \), the lowest capacity/demand ratio of \( n \) equipment items will be greater than unity may be estimated by requiring

\[
P_F = 1 - (1-\gamma)^{(n+1)} = \Phi(-u_0),
\]

or

\[
-u_0 = \Phi^{-1}\{1 - (1-\gamma)^{(n+1)}\}.
\]

Since \( u_0 = \ln(X)/\beta_X \),

\[
X = C/D = e^{u_0\beta_X}.
\]

If the median demand, \( D \), is estimated as \( D = D_{\text{ave}} = RS \), then the capacity associated with 95% confidence is given by

\[
C_{95} = RS e^{u_0\beta_X}.
\]

The High Confidence Low Probability of Failure (HCLPF), or 95% confidence of less than a 5% failure probability, is given by the 5% capacity level, or

\[
C_{HCLPF} = RS e^{u_0\beta_X - 1.645\beta_C} = RS F_K.
\]

where the factor \( F_K = e^{u_0\beta_X - 1.645\beta_C} \) is the reduction or knockdown factor applied to the reference capacity spectrum, i.e., EES, to achieve a HCLPF capacity value.

Taking \( \beta_D = 0.3 \) and \( \beta_C = 0.4 \) as representative lognormal standard deviations for spectral demand and capacity, then \( \beta_X = 0.5 \), and the following tabulation of capacity/demand ratios for a confidence coefficient \( \gamma = 0.95 \), or a 95% confidence level, for equipment survival is obtained for class group sizes ranging from 60 to 15.

<table>
<thead>
<tr>
<th>n</th>
<th>( P_F )</th>
<th>(-u_0)</th>
<th>( X = C/D )</th>
<th>( F_K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>0.047924</td>
<td>-1.66533</td>
<td>2.299</td>
<td>1.191</td>
</tr>
<tr>
<td>50</td>
<td>0.057048</td>
<td>-1.58005</td>
<td>2.203</td>
<td>1.141</td>
</tr>
</tbody>
</table>
A class group size of 30 is the minimum number of items necessary to demonstrate that the reference capacity spectrum, i.e., EES, without applying a reduction factor, represents a conservative estimate of the HCLPF capacity.

**True Median Capacity**

The development outlined above provides an estimate of the population mean, \( \ln(C) \), which, for high levels of confidence, will be conservative (i.e., low) compared to the true population mean. The situation, as a set of \( n \) observations of no damage for the demand level recorded or estimated for each observation, may be interpreted as a sample taken from a large population of equipment meeting the attribute limits or caveats of the equipment class per QR-A7421. Estimating the sample mean capacity, or \( \ln(C) \), for which the conservatism is removed would provide an estimate of the true median capacity of the equipment to be used in risk-informed seismic evaluations of equipment.

One method of achieving this capacity estimate is to consider the HCLPF values computed above, RS \( F_K \), as one-sided lower tolerance limits based on the sample size and sample mean value. This may be represented by

\[
\ln(C_{np\theta}) = \ln(C) - k_{np\theta},
\]

where \( C_{np\theta} \) is the lower tolerance limit such that the probability is \( p \) that at least a proportion \( \theta \) lies below \( C_{np\theta} \) (or a proportion \( 1-\theta \) lies above \( C_{np\theta} \)), and where \( k_{np\theta} \) is the tolerance factor based on \( p \), \( \theta \), and sample size, \( n \).

In general, for the case of a known (or assumed) standard deviation (Hald, 1952),

\[
k_{np\theta} = -\Phi^{-1}(\theta) + \Phi^{-1}(p)/(n)^{1/2}.
\]

If \( p = 0.95 \) and \( \theta = 0.05 \), and \( C_{np\theta} = C_{HCLPF} = RS F_K \), then

\[
\{C/RS\}_{tol} = F_K e^{k_{np\theta}},
\]

and the following tabulation is obtained using the prior results for \( F_K \):
Another estimate of the mean spectral capacity may be achieved by noting that the HCLPF capacity may be approximated by the 1% value \(\Phi^{-1}(0.01) = -2.326\) of capacity (Kennedy, 1999):

\[
C_{\text{HCLPF}} \approx C e^{-2.326\beta C}.
\]

Again, let \(C_{\text{HCLPF}} = RS FK\). Then

\[
\{C/RS\}_{1\%} = FK e^{2.326\beta C},
\]

resulting in the alternate tabulation:

<table>
<thead>
<tr>
<th>n</th>
<th>FK</th>
<th>(e^{2.326\beta C})</th>
<th>{C/RS}_{1%}</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>1.191</td>
<td>2.102</td>
<td>2.503</td>
</tr>
<tr>
<td>50</td>
<td>1.141</td>
<td>2.119</td>
<td>2.418</td>
</tr>
<tr>
<td>40</td>
<td>1.081</td>
<td>2.142</td>
<td>2.317</td>
</tr>
<tr>
<td>35</td>
<td>1.046</td>
<td>2.158</td>
<td>2.258</td>
</tr>
<tr>
<td>30</td>
<td>1.006</td>
<td>2.177</td>
<td>2.190</td>
</tr>
<tr>
<td>25</td>
<td>0.959</td>
<td>2.202</td>
<td>2.113</td>
</tr>
<tr>
<td>20</td>
<td>0.903</td>
<td>2.237</td>
<td>2.021</td>
</tr>
<tr>
<td>15</td>
<td>0.833</td>
<td>2.288</td>
<td>1.907</td>
</tr>
</tbody>
</table>

Viewing these two mean capacity estimates as upper, \{C/RS\}_{1\%}, and lower, \{C/RS\}_{tol}, bounds, the median capacity may be estimated by the geometric average of the two bounds:

<table>
<thead>
<tr>
<th>n</th>
<th>L={C/RS}_{tol}</th>
<th>U={C/RS}_{1%}</th>
<th>(UL)^{1/2}</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>2.503</td>
<td>3.020</td>
<td>2.635</td>
</tr>
<tr>
<td>50</td>
<td>2.418</td>
<td>2.894</td>
<td>2.525</td>
</tr>
<tr>
<td>40</td>
<td>2.317</td>
<td>2.742</td>
<td>2.378</td>
</tr>
<tr>
<td>35</td>
<td>2.258</td>
<td>2.653</td>
<td>2.321</td>
</tr>
<tr>
<td>30</td>
<td>2.190</td>
<td>2.551</td>
<td>2.226</td>
</tr>
<tr>
<td>25</td>
<td>2.113</td>
<td>2.433</td>
<td>2.183</td>
</tr>
<tr>
<td>20</td>
<td>2.021</td>
<td>2.291</td>
<td>2.096</td>
</tr>
<tr>
<td>15</td>
<td>1.907</td>
<td>2.113</td>
<td>1.990</td>
</tr>
</tbody>
</table>
Sensitivity to $\beta_C$

The sensitivity of $\beta_C$ on the results is checked for $n=30$:

<table>
<thead>
<tr>
<th>$\beta_D$</th>
<th>$\beta_C$</th>
<th>$\beta_X$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.450</td>
<td>0.54</td>
</tr>
<tr>
<td>0.3</td>
<td>0.400</td>
<td>0.50</td>
</tr>
<tr>
<td>0.3</td>
<td>0.335</td>
<td>0.45</td>
</tr>
</tbody>
</table>

The sensitivity of the results to the uncertainty $\beta_C$ is small.

**Conclusion**

The technical basis is provided for the minimum number of independent equipment items to define an equipment class using earthquake experience as specified in section QR-A7422 and the reduction factors, given in Table QR-A7422-1, required for reducing the EES when a smaller number of independent items are used to define an equipment class. The reduction factors per Table QA-A7422-1 are a conservative (lower) round off of the reduction factors calculated in this paper. Also the results were shown not to be very sensitive to the assumed log-normal standard deviation of capacity.

**References**
